Sediment transport and morphodynamics

Dr. Sergio Maldonado
CENV3066 Environmental Hydraulics
University of Southampton
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Contents

1 Introduction

Students of open channel hydraulics derive much of their knowledge from the (often implicit) assumption that the bed of the channel is fixed, such that it affects the flow[1] but is not in turn affected *by* the flow. However, the study of natural rivers and streams (the examples of open channel by excellence), as well as that of estuaries and the coast, confronts us with another, more complex reality. Natural open channel beds are composed of loose material (sediment) that can be dislodged and transported by the flow, thus leading to erosion/deposition processes and **sediment transport**[2]. Moreover, the erosion and accumulation of sediment in different parts of the bed implies an evolving **bed morphology**[3]. But the water flow is in good measure governed by the bed (a boundary), which in turn evolves because of the flow itself! This gives raise to a feedback between the water flow and the bed over which it flows. We call this **morphodynamics**: the dynamics of a changing bed.

The study of sediment transport and morphodynamics may appear as a daunting task due to the inherent complexity and randomness involved. After all, sediment comes in all shapes, sizes and layout (on the bed), and it is usually subject to turbulent flow, which, as we just learnt, is itself a complex and random phenomenon. Because of this, the preferred tool to approach this problem has historically been empiricism, understood here as the method that advocates solution to complex problems based on experimental, approximate findings, rather than rigorous theoretical derivations based on first principles[4]. In this sense, our current knowledge of pure-water flows is centuries ahead of our understanding of the mechanics of sediment transport and morphodynamics. The latter are very active areas of research today. That being said, the past few decades have seen an increasing interest and, with it, body of knowledge in this field. Thus, in these notes, I shall try to introduce you to this topic by presenting the 'conventional', well-established approach while also pointing out, when relevant, what its limitations are and what the state-of-the-art research has to say about it.

2 Initiation of motion

The most basic question regarding the motion (or transport) of sediment is arguably this: when will a grain of sediment resting on the bed start to move? This apparently simple question represents one of the many unsolved problems in this field. At first sight, this may seem difficult to believe. After all, this is well-established classical mechanics (not modern theoretical physics). We *should* know what forces and equations govern the motion of this initially resting particle, right? Right, the problem is not that we ignore the origin of the forces that govern the motion of this initially resting grain, but rather that we cannot pragmatically obtain information about said forces – i.e.

[1] Essentially, the bed is treated as a boundary condition, as discussed previously in this module.

[2] You can probably guess what the practical implications of this are: e.g. scour around structures, dam siltation, coastal erosion, and so on.

[3] From Greek *morphe*, 'form'. Thus, in this context, morphology refers to the form or shape of the bed (also known as **bathymetry**).

[4] There is of course nothing wrong with experimentation itself, which is actually the core of the scientific method. Empiricism in this context refers to the rough analysis of experimental results without necessarily accounting for the physics behind the patterns observed; for example, a best-fit curve to a set of points (which, as we will see later, is a common tool in sediment transport research).

we cannot estimate them accurately in practice.

Consider a single sediment *particle*[5] resting on top of other similar particles, which is what the bed actually is. Essentially, the forces acting on this target particle are those caused by gravity (its weight), the contact with other particles and the *net* force exerted by the fluid. The two former are relatively easy to quantify, but how about the fluid force? Strictly speaking, the net fluid force would include (i) forces due to friction between the fluid and the surface of the particle (skin friction), and (ii) forces due to pressure differences at different points on the surface of the particle (form drag and lift)[6]. Mathematically, this equates to saying that the net hydrodynamic force on the particle is nothing but the integral of pressure and shear stresses over the surface of the particle (see fig. 1). Can you imagine doing this for every single sediment grain resting on the bed of a river?

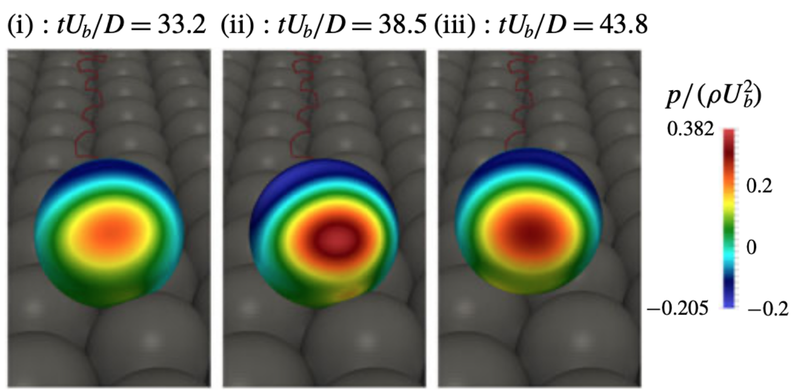

Figure 1: Snapshots of (normal) pressure on the surface of a resting sphere at different times during a turbulent flow event. The integral of this pressure over the surface of the sphere gives you the main hydrodynamic forces acting on the sphere: the sum of lift and drag. A similar integral for the tangential (shear) stresses gives you the skin friction. [Taken from Yousefi *et al.* (2020) *J. Fluid Mech.* **893** A24.]

Obviously not. Therefore, a different, more pragmatical approach ought to be adopted. To this end, let us 'zoom out' and look at the problem from a more 'macroscopic' (or *bulk*) scale. At a larger scale, you can think of the flow as exerting a *bulk* shear stress on the bed (which is in turn, the resistance 'felt' by the flow). This shear stress, τ, multiplied by some representative bed area, A_τ, translates into a shear force. At the particle scale, we can think of this shear force as the sum of drag forces acting on all the particles within A_τ (see fig. 2). The representative area, A_τ, is related to some projection of the top-most erodible particles, and should therefore go like the square of some characteristic length. Real sediment particles come in all sort of shapes, but most of the times they can be treated as sufficiently 'round' so as to speak of a representative diameter D, such that the representative area is $A_\tau \propto D^2$ (again, see fig. 2). Therefore, the shear force $\tau A_\tau \propto \tau D^2$.

The hydrodynamic shear force is trying to erode the top-most particles on the bed[7]. The coun-

[5]In this field, it is common to use interchangeably the terms *grain* and *particle*. A physicist may be utterly disturbed by this!

[6]If we are really strict, we should include buoyancy here as well, but buoyancy is typically accounted for by using a *modified weight* of the particle.

[7]To be more rigorous, there are also lift forces involved, but it is commonly accepted that streamwise directed drag (leading to shear) is the main driver of bed erosion.

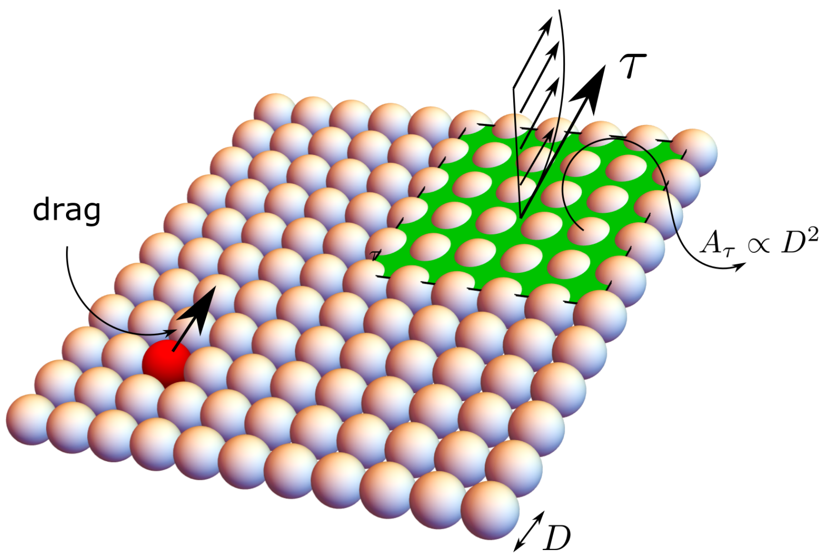

Figure 2: The fluid bulk shear stress τ, acting on A_τ, translates into a shear force applied to all the particles within A_τ (intersected by the green plane). This force can also be thought of as the sum of drag forces acting on each particle (see e.g. red sphere) within A_τ. The value of the latter will naturally depend on the number of particles involved, each of diameter D, but we know that in general $A_\tau \propto D^2$.

terweight to this destabilising force is related to the other forces we have previously discussed – weight and contact forces – which ultimately relate to the particles' submerged weight[8]. For a single particle, its submerged weight is $(\rho_s - \rho)gV_p$, where g is the gravitational acceleration, V_p is the volume of the particle, and ρ_s and ρ are the densities of the particle and water, respectively. Of these parameters, only V_p is problematic, for, again, how can we estimate accurately the volume of every single particle on the bed? (density tends to be remarkably constant for a given type of sediment) However, we know that in any case it should be expected that $V_p \propto D^3$ (in the case of perfect spheres, obviously $V_p = \pi D^3/6$). The challenge is to find a *critical* value of τ – let us call it the **critical bed shear stress** τ_c – that is capable of overcoming the stabilising forces related to the particle's submerged weight. To this end, the power of dimensional analysis is exploited, as with many other complex problems in hydraulics. If there is an important parameter governing the initiation of motion, it is likely to involve the ratio of the destabilising force (due to the bed shear stress) and the stabilising forces (related to the submerged weight of the erodible particles)[9]. This was probably the line of reasoning that led A. Shields, an American engineer studying in Germany in the 1930's, to show **empirically** that the non-dimensional number combining these two forces[10],

$$\theta_c \equiv \frac{\tau_c}{(\rho_s - \rho)gD}, \tag{1}$$

actually does well in predicting when the bed will start to erode (or what is the same, the top-most particles will start to move) for a given (type of) Reynolds number, in turn defined as

$$\mathrm{Re}_* \equiv \frac{u_{*c}D}{\nu}, \tag{2}$$

[8]i.e. the particle's actual weight minus buoyancy force.

[9]Think of the analogy with the basic mechanics problem of static friction. What is the static friction coefficient if not the ratio of the critical destabilising force to the stabilising forces (which are related to weight)?

[10]We get this number by dividing $\tau_c D^2$ by $(\rho_s - \rho)gD^3$. Show that this is indeed non-dimensional.

where ν is the kinematic viscosity of water and $u_{*c} \equiv \sqrt{\tau_c/\rho}$ is the critical shear velocity (so called because it is, by definition, associated with the critical shear stress we are looking for). In other words, Shields found that the threshold of erosion he observed experimentally was (relatively) well explained by a curve $\theta_c = f(\mathrm{Re}_*)$. Today we call this curve, naturally, **Shields curve**, which is a tool widely used by engineers and scientists alike. The variable θ_c is often referred to as **Shields parameter**, **entrainment function** or the non-dimensional critical bed shear stress.

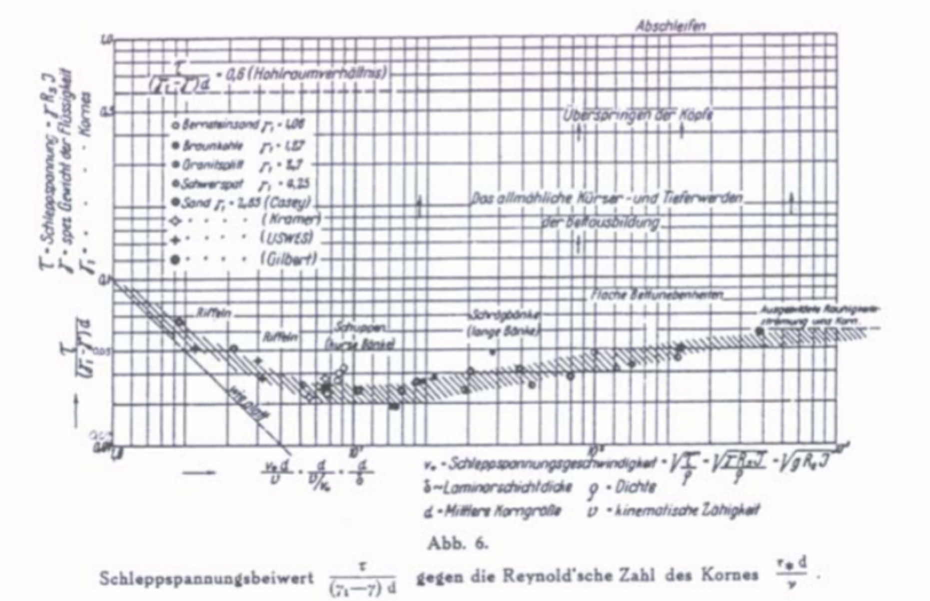

Figure 3: Original curve for initiation of motion from Shields' PhD thesis.

However, this curve presents important limitations. As can be seen in the original plot in Shields' PhD thesis (fig. 3), Shields actually reports a band, not a curve, which implies that Shields himself was well aware of the approximate nature of his findings. Moreover, via u_{*c}, the variable τ_c appears in both sides of the equation $\theta_c = f(\mathrm{Re}_*)$, making it an implicit function, which complicates its use. This caveat, however, is remedied by plotting θ_c, not as function of Re_*, but as function of another non-dimensional variable dependent on the particle's characteristics and fluid's viscosity; namely, the non-dimensional particle diameter:

$$D_* \equiv D\left[\left(\frac{\rho_s}{\rho} - 1\right)\frac{g}{\nu^2}\right]^{1/3}. \tag{3}$$

This leads to a modified version of Shields curve (see fig. 4), which is the one actually used today. There are a few interesting remarks about this curve. For instance, we observe that for large values of D_* (say $D_* \gtrsim 20$), θ_c increases with D_*; that is, the larger (and thus heavier) the particles, the larger the shear stress required to mobilise them. This makes sense. But for small values of D_* (say $D_* \lesssim 10$) we observe the opposite: θ_c increases for smaller values of D_*. Why should we expect to require larger values of shear stress for smaller (and thus, lighter) particles? One reason is that for very small particles, such as clay, additional forces of electrostatic nature play a major role in stabilising the bed[11]. We call this sediment **cohesive**. The mechanics of cohesive sediment are

[11] Can you think of another potential reason (perhaps related to the viscous sublayer)?

very interesting but quite different from those of larger, **non-cohesive** sediments. In this module, we will only address the latter.

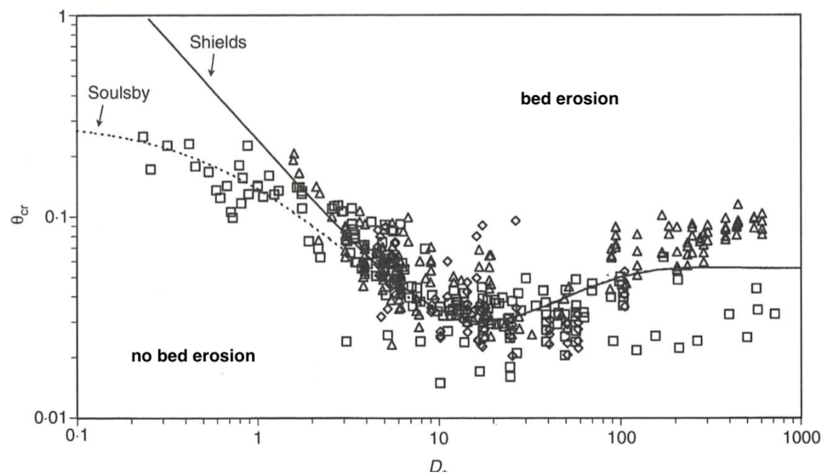

Figure 4: Modified Shields curve ($\theta_{cr} =)\theta_c = f(D_*)$. Symbols represent various experimental data, while lines represent curve fits to data (these are only two out of *many* such empirical fits). I leave it up to you to judge how well or bad these curves fit the experimental data. [Modified from Soulsby, R. *Dynamics of marine sands.* Thomas Telford, 1997.]

Another important feature of Shields curve is its evident uncertainty. Note that for a given value of D_*, one can easily expect an uncertainty in the predicted θ_c of about one order of magnitude(!). There are several reasons for this. Mainly, you may have noticed that we have avoided all mention of time and space averages of variables, contrary to what we did when studying Open Channel Hydrodynamics. For instance, when we discussed above the hydrodynamic force acting on the bed particles, did we mean instantaneous or time-averaged force? Textbooks and practitioners will often quietly ignore this point, which is of great relevance. Sediment particles on the bed are typically subject to turbulent (and thus fluctuating) flow. Hence, if your calculation of τ is based on, say, time-averaged flow velocities, you may be ignoring instantaneous fluctuations (or pulses) which can be large enough to dislodge the top-most particles – i.e. you will be underestimating the flow capability to erode the bed. Some recent research has proved that indeed the impulse[12] of hydrodynamic forces, and not their time-averaged values, represents a more accurate criterion for predicting whether erosion will occur. However, this research is at an early stage and it is still difficult to see how concepts such as impulse could be included in engineering practice[13].

So far we have also avoided mention of the channel inclination, which is a very important characteristic of open channel flows. The effect of the bed slope manifests itself in the component of the particle's weight opposing motion, so the definition of θ_c in eq. (1) implicitly assumes that its effect is negligible. This is only true when the bed is horizontal or nearly horizontal, which actually happens to describe well many natural rivers and estuaries. However, for steep channels (as in mountain rivers) or for river banks or certain beaches (and for the sake of generality), the effect of the bed slope may be included in a simple way by adding vectorially the additional component of

[12] i.e. time integral of force

[13] Can you see why?

the weight, yielding:

$$\frac{\tau_{\beta c}}{\tau_c} = \frac{\cos\psi \sin\beta + \left(\cos^2\beta \tan^2\phi - \sin^2\psi \sin^2\beta\right)^{1/2}}{\tan\phi}, \tag{4}$$

where $\tau_{\beta c}$ is the critical shear stress on a channel of slope angle β (with respect to the horizontal), τ_c is the critical shear stress on a horizontal bed (i.e. the one obtained from Shields curve), ψ is the angle that the flow makes with the upsloping direction (see fig. 5), and ϕ is the angle of repose (the angle at which sediment avalanches under zero flow). When the flow is directed laterally across the slope ($\psi = \pm 90°$, as in river banks), the above equation reduces to

$$\frac{\tau_{\beta c}}{\tau_c} = \cos\beta \left(1 - \frac{\tan^2\beta}{\tan^2\phi}\right)^{1/2}, \tag{5}$$

whereas for the case of flow being aligned with the bed slope ($\psi = 0°$), we have[14]

$$\frac{\tau_{\beta c}}{\tau_c} = \frac{\sin(\beta + \phi)}{\sin\phi}. \tag{6}$$

Estimating whether the top-most grains will be eroded is just the beginning of the problem, for once they have been mobilised, they are then transported by the flow along the bed, and they do so in different modes, as we shall describe next.

3 Bedload transport

When the shear stress exerted by the flow on the bed exceeds the threshold of motion, sediment starts to be transported along the channel. If τ is not much larger than τ_c, the motion of sediment will be confined to the region near the bed, and particles will move either by *rolling, sliding or saltating* (hopping) along the bed. We call this type of sediment motion **bedload**. Bedload transport is typically associated with relatively large sediments such as gravel and coarse sand[15].

Quantifying the amount of sediment that is transported by a current per unit time (i.e. the **sediment transport rate**) is very important in hydraulic engineering (e.g. for estimating dam siltation or shoreline retreat rates), but is also a very challenging task due to all the complexities discussed so far. This problem, more than any others, has been heavily dominated by empirical approaches that simply look for a practical formula (often resulting from best fits to experimental data) that works to a satisfactory level in field applications. It is likely that empirical formulae will remain the standard tool employed by the hydraulic engineer for many years to come, but there is no much point in memorising the dozens of existing empirical relations – to this end, you may always consult manuals and textbooks on the matter[16]. However, many of these formulae are

[14] Arrive at this equation by considering an analogy with the Mechanics 101 problem of a box on a ramp.

[15] Note that by 'relatively' we mean in relation to the flow conditions. For a very slow moving fluid, sand may move as bedload, whereas the same sediment might be entrained into suspension (see §4) by a faster moving flow.

[16] For example, the book *Dynamics of marine sands* by Soulsby, R. (Thomas Telford, 1997), from which several figures are taken here, is an excellent go-to book for practical issues about sediment transport in the coastal environment.

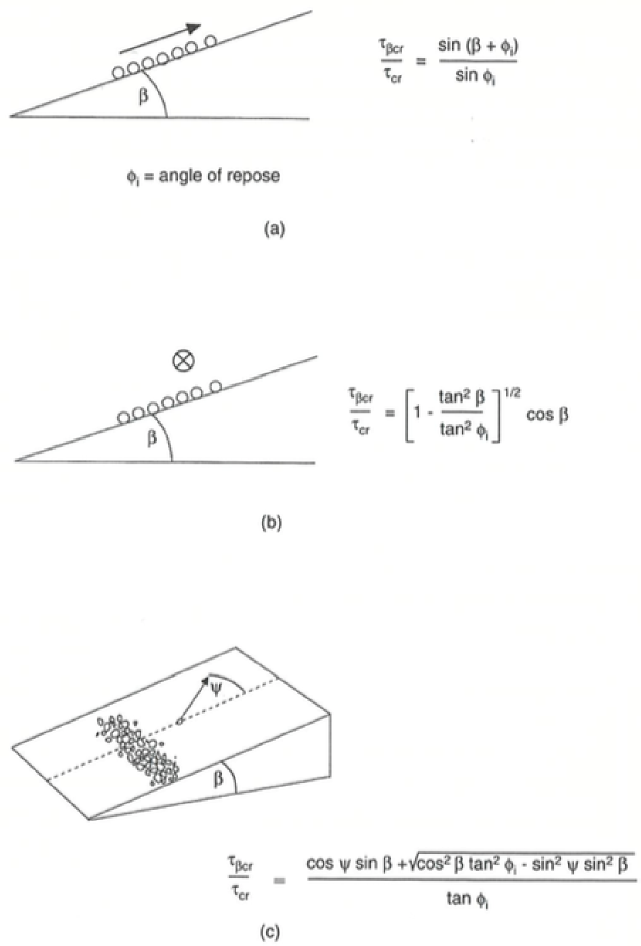

Figure 5: Threshold of motion on sloping beds. [Taken from Soulsby, R. *Dynamics of marine sands.* Thomas Telford, 1997.]

based, at least to some degree, on certain physical principles which are worth discussing since they provide us with important insights into the problem.

In the previous section we discussed Shields criterion, which employs the bed shear stress to predict whether a given bed will erode or not. It is thus natural to extend this criterion to the prediction of sediment transport rates; i.e. to estimate the volumetric bedload transport rate, q_b, as some function of τ. This idea goes back to the 19^{th} century French engineer Paul Du Boys, whose work served as the basis for the development of further theories of bedload. One such theory argues that q_b may be expressed as some power series[17] of τ; namely,

$$q_b(\tau) = K_0 + K_1\tau + K_2\tau^2 + K_3\tau^3 + ... \tag{7}$$

for $\tau \geq \tau_c$. Now, if terms of order > 2 are neglected[18], and the following two self-evident conditions are invoked: $q_b = 0$ if $\tau < \tau_c$ and $q_b = 0$ if $\tau = \tau_c$; then it follows that $K_0 = 0$, $K_1 = -K_2\tau_c$,

[17]After all, any function $f(x)$ can be locally approximated by a power series of x (Taylor expansion).

[18]We will discuss in class why it is sensible to do so, but for now suffice to say that in many problems in nature, the dominant (or leading-order) terms are the low-order ones.

yielding[19]

$$q_b = K_2(\tau - \tau_c)\tau, \tag{8}$$

The problem then reduces to determining K_2. This is typically done via empirical methods – i.e. fit the above curve to a set of experimental data and 'tweak' K_2 until you get a good fit. However, an ever growing body of empirical evidence shows that, actually, a constant value of K_2 does not yield an accurate expression for q_b. Nonetheless, the equation above does convey an important message: in general, we expect the bedload rate to increase (non-linearly) with some combination of bed shear stress and **excess bed shear stress** (i.e. the difference $\tau - \tau_c$). In this case, when $\tau \gg \tau_c$, we can expect $q_b \sim \tau^2$.

Another non-linear relationship between q_b and τ can be derived theoretically by following on the steps of the prominent 20$^{\text{th}}$-century English sedimentologist (and soldier of the British Army), R. A. Bagnold. Bangold's approach, in essence, was to think about the work done by the flow on the bed particles when transporting them – more specifically, on the rate at which this work is done (i.e. power). The rate at which work is done on an object is given by the (dot) product of the force doing the work and the velocity of the object. In a given area of bed, the force mobilising the bed particles will depend on τ, and the corresponding velocity of the bedload particles will be close to some near-bed velocity. As discussed towards the end of Open Channel Hydrodynamics, τ in turbulent flows varies as U^2, where U is the depth-average velocity. Also, any near-bed velocity, $u(z \to z_b)$, can be seen as a fraction of U; i.e. $u(z \to z_b) \sim U \sim \tau^{1/2}$. So the available 'flow power' near the bed will vary as $\tau U \sim \tau^{3/2} \sim U^3$. Of course, Bagnold pointed out that not all of this power will be converted into motion of the bedload particles (think of it as a sort of 'transport efficiency'). But still, for our purposes here, that remark does not change the fact that we expect the trend $q_b \sim \tau^{3/2}$ to hold true (note, the exponent is different than in Du Boys' approach). Fig. 6 shows some lab data fitted with a curve of the form $q_b \propto (\tau - \tau_c)^{3/2}$, which shows that the scaling[20] prediction from Bangold's approach is good[21].

In addition to the approaches discussed above, there exists a plethora of empirical and semi-empirical formulations for bedload. The vast majority of them may be expressed in the general form[22]:

$$q_b = F\left(\tau^{m1} - \tau_c^{m2}\right)^{n1}\left(\tau^{m3} - \tau_c^{m4}\right)^{n2}\tau^{n3}, \tag{9}$$

where $m1, m2, m3, m4, n1, n2$ and $n3$ are all real constants. Table 1 shows the values of said constants for some popular bedload formulae. The coefficient F, on the other hand, may or may not be taken as a constant. For instance, for the formulae by Meyer-Peter and Müller (1948) and Fernández Luque and van Beek (1976), F is taken as a constant; but for van Rijn (1984), for

[19] Show this .

[20] In the absence of accurate, theoretical predictions giving absolute values of q_b, scaling arguments are very valuable. The symbol $\sim$ can be read as 'varies as', 'goes like', or 'scales with', and it denotes large values of the variable involved. For example, $y \sim x^2$ may be read: 'y varies with the square of x, at least for large values of x'. In these notes, when I use the other symbol $\propto$ (proportional to), I want to say that the only thing missing is a multiplicative factor (when using $\sim$ an additive constant may or not be involved). In other words, $y \propto x^2$ implies $y = Kx^2$, while $y \sim x^2$ may also apply to $y = Kx^2$, but also to $y = Kx^2 + Mx + N$. Note, however, that the use of $\sim$ and $\propto$ varies from source to source in the literature.

[21] The scaling $q_b \sim \tau^{3/2}$ implies that $\tau \gg \tau_c$. Indeed, for conditions very close to the threshold of motion (i.e. $\tau \to \tau_c$), very different scaling laws apply. The reasons are still unclear – this is a very active area of research.

[22] Although, usually, they will be expressed in non-dimensional form; i.e. θ instead of τ, and a similar non-dimensional version of q_b, typically denoted Φ.

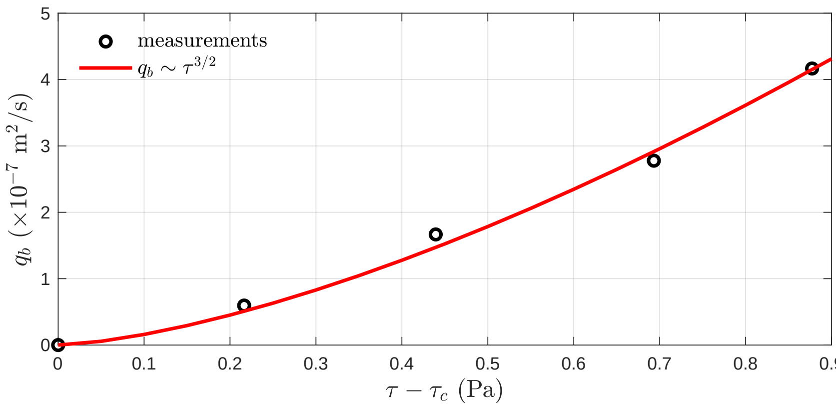

Figure 6: Volumetric bedload transport rate (per unit channel width), q_b, against excess bed shear stress, $(\tau - \tau_c)$. Symbols represent experimental data, while red line is a best-fit curve of the form $q_b = A_{bf}(\tau - \tau_c)^{3/2}$, where A_{bf} is a fitting parameter.

Table 1: Values for exponents in eq. (9), for different bedload formulations. Values not shown (since they are not relevant) when $n1 = 0$ or $n2 = 0$.

Formula	$m1$	$m2$	$m3$	$m4$	$n1$	$n2$	$n3$
Meyer-Peter and Müller (1948)	1	1	-	-	3/2	0	0
Bagnold (1963)	1	1	-	-	1	0	1/2
van Rijn (1984)	1/2	1/2	-	-	2.4	0	1/2
Yalin (1963)	1	1	-	-	1	0	1/2
Ashida and Michiue (1972)	1/2	1/2	1	1	1	1	0
Wilson (1966)	-	-	-	-	0	0	3/2
Nielsen (1992)	1	1	-	-	1	0	1/2
Fernández Luque and van Beek (1976)	1	1	-	-	3/2	0	0
Soulsby (1997)	1	1	-	-	1	0	1/2

instance, F is a function of the water depth, sediment diameter and the bed friction coefficient. As mentioned previously, this topic is notoriously dominated by empiricism. The actual formulae presented in Table 1 and their corresponding ranges of validity may be found in their original publications or in popular textbooks on the topic.

Several other approaches have been adopted to try to understand and quantify bedload. One such approach worth mentioning was put forward by Hans Albert Einstein[23] in the mid-20$^{\text{th}}$ century, and employed probabilistic methods to describe bedload. Although quite original, his theories, however, have not become terribly popular among practitioners to this day.

Exercise: The data in table 2 was used to generate fig. 6. Select any three formulae from table 1 and fit them to the aforementioned data. Comment on the accuracy of each expression.

[23] Son of *the* Albert Einstein, Hans became a prominent hydraulic engineer known for his pioneering work on sediment transport. A (most likely apocryphal) legend claims that, after listening to Hans explain his theory of bedload transport in rivers, Albert Einstein asked him to stop because all that was too difficult to grasp! (In reality, Albert may have even helped Hans work out his probabilistic theory of bedload.)

Table 2: Data used to generate fig. 6

τ (Pa)	q_b ($\times 10^{-7}$ m^2/s)
0.781	0.0
0.997	0.595
1.220	1.667
1.474	2.778
1.658	4.167

NOTE: This exercise is merely for practice, but when selecting empirical expressions for real applications beware that expression such as those from table 1 are usually limited to the range of experimental parameters and conditions they were derived for; for example: subcritical flow, $D < 0.23$ mm, $1 < \tau/\tau_c < 5$, etc. These expressions can be uncertain as they are, but using them outside their range of validity can further amplify such uncertainty. So, always check the range of validity for each expression!

4 Suspended load

When the bed shear stress far exceeds the threshold of motion[24], sediment entrains suspension mode. This means that rather than being confined to the near-bed region, sediment is transported long distances by the flow in the region of the latter away from the bed (i.e. it is *suspended*). At any given time, gravity is trying to bring each grain of sediment down. Thus, for the latter to remain in suspension, an upward force must be acting upon it. To understand this better let us perform a simple mental experiment. Imagine a vertical tube filled with still fluid. Then drop a grain of sediment inside it. If gravity alone acted upon the grain, the latter would accelerate uniformly, such that its velocity would continuously increase with time (up to infinity if given infinite time!). However, in reality, the grain also experiences (flow-induced) drag, which opposes motion. Since the drag force increases with the square of the grain velocity (relative to the fluid), there must be a *terminal velocity* for which drag force is equal to the particle's (submerged) weight, such that particle's acceleration eventually ceases to exist (the particle continues to fall but with constant velocity). This velocity is called, unsurprisingly, the **fall or settling velocity**, w_f. And, as mentioned, it can be obtained from the equilibrium condition where drag force and particle's weight balance each other; namely:

$$\underbrace{\frac{1}{2} C_D \rho A_s w_f^2}_{\text{drag}} = \underbrace{(\rho_s - \rho) g V_p}_{\text{subm. weight}}, \tag{10}$$

where A_s is the projected area of the particle normal to the flow (naturally, for a spherical particle of diameter D, $A_s = \pi D^2/4$); V_p is, as before, the particle's volume; and C_D is the drag coefficient. For the sake of generality, in the equation above, w_f should be replaced by the particle velocity *relative to the flow*[25]. The important message here is that, due to drag, **the particle eventually attains a terminal velocity relative to the fluid**. So, in the second stage of our mental experiment, let us now connect the tube to a pipeline powered by a pump, such that a flow velocity

[24] This can happen either because of a very fast flow, very fine (light) sediment, or both.
[25] which is obviously equal to w_f for the particular case of still fluid.

is induced upwards (opposite to the motion of the falling particle). Let us call this velocity, w; then the particle will appear to fall at a velocity $(w_f - w)$ (seen from an stationary frame of reference – i.e. you watching the experiment). Something interesting happens when we increase the pump power to the point where $w = w_f$ (recall, w is directed opposite to w_f): the particle seems not to fall at all – it is *suspended*.

The key point from the above example is the following: **for sediment particles to remain in suspension, upwards forces induced by the vertical component of the flow velocity must act upon them**. But if the flow is streamwise directed as in most rivers, how can there be upward forces induced by the flow? The answer is to be found in turbulence. Recall that a signature of turbulent flow is that the velocity field presents fluctuations in time and (all of the three coordinates of) space. Even for a flow where the *mean* vertical component of the velocity is null (i.e. $\langle w \rangle = 0$), there exist non-zero fluctuations, w', which are responsible for the suspension of sediment. Furthermore, w' increases with increasing turbulence[26], which is in agreement with our earlier statement that suspended load is associated with faster flows than those responsible for bedload.

In general, the volumetric concentration of suspended sediment (i.e. the volume of sediment suspended per unit volume of fluid) varies from place to place (and time) within the fluid, and it is typically higher near the bed than at the free surface[27]. Further insight into the distribution of suspended sediment concentration can be gained by realising that the latter behaves (to a good approximation) as a scalar physical quantity (just like heat or salinity) that is *transported* by the fluid, and can thus be modelled by the **advection-diffusion equation**. The advection-diffusion equation[28] is derived from conservation principles and simply states that any scalar quantity, S, is transported by a fluid via two main mechanisms: advection and diffusion. Thus, locally, the temporal variation of S is given by:

$$\frac{\partial S}{\partial t} = \underbrace{-\nabla \cdot (\mathbf{v} S)}_{\text{advection}} + \underbrace{\nabla \cdot (\mathbf{D} \nabla S)}_{\text{diffusion}} \tag{11}$$

(if no sinks or sources of S are considered), where $\mathbf{v}$ is the velocity at which the quantity of interest is moving[29], and $\mathbf{D} = (D_x, D_y, D_z)$ is the diffusion (or dispersion[30]) coefficient (with components for each Cartesian direction), which depends inherently on the quantity being diffused and the medium where the diffusion occurs. Consider that S represents the suspended sediment concentration, $C = C(x, y, z, t)$; and $\mathbf{v} = (u_s, v_s, w_s)$ is the velocity vector field of the water-sediment mixture (or 'dispersoid'), which is divergence-free[31], i.e. $\nabla \cdot \mathbf{v} = 0$; and replace $\mathbf{D}$ with the **sediment diffusion**

[26] Recall, w' would be equal to zero only in laminar flows.

[27] **Q:** Can you see why?

[28] Also known as convection-diffusion equation or scalar transport equation.

[29] If S represented, for instance, heat, then $\mathbf{v}$ would denote the fluid velocity field. But, as we will see soon, when S relates to 'massive' particles such as those of sediment, $\mathbf{v}$ ought to represent the velocity of the mixture as a whole.

[30] see footnote 32.

[31] A couple of important points here. First, we are tracking the evolution of the concentration C, which is by definition a mixture of water and sediment (it is the volume of sediment to be found in a given volume of water; or rather, the ratio of said volumes). It is thus the velocity of this mixture what concerns us. Notice that sediment *must* move at a different (lower) speed than the *driving* flow; in fact, we can say that $\mathbf{v} = \mathbf{v}_s C + \mathbf{v}_f (1 - C)$, where $\mathbf{v}_s$ and $\mathbf{v}_f$ are the sediment and fluid velocities, respectively. Second, remember that a vector field has a non-zero divergence only at *sources* or *sinks*. In this case, the bed acts as a *source* of sediment. However, we treat the bed as a *boundary* of our fluid domain, and so the divergence-free condition continues to apply to $\mathbf{v}$.

coefficient $\mathbf{D}_s = (D_{sx}, D_{sy}, D_{sz})$. Then the above equation can be written in extended form as[32]:

$$\frac{\partial C}{\partial t} + u_s \frac{\partial C}{\partial x} + v_s \frac{\partial C}{\partial y} + w_s \frac{\partial C}{\partial z} = \frac{\partial}{\partial x}\left(D_{sx}\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{sy}\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_{sz}\frac{\partial C}{\partial z}\right). \tag{12}$$

Solution to the above equation, with corresponding boundary and initial conditions, tells us what the sediment concentration, $C(x, y, z, t)$, will be at any point (in the 3D domain) and time. But, frequently, we are interested in flows that can be treated, at least approximately, as steady and uniform, such that C varies only in the vertical direction. This means $\partial C/\partial t = \partial C/\partial x = \partial C/\partial y = 0$, such that we are left with $\partial C/\partial z = dC/dz$ and

$$w_s \frac{dC}{dz} = \frac{d}{dz}\left(D_{sz}\frac{dC}{dz}\right). \tag{13}$$

Now, in the steady-state condition invoked, we can replace w_s with the fall velocity (w_f) discussed above; in particular, we have $w_s = -w_f$ (negative sign due to downwards motion). Since w_f does not depend on z, we can integrate both sides to arrive at[33]

$$D_{sz}\frac{dC}{dz} = -w_f C. \tag{14}$$

The above equation tells us that in steady, one-dimensional, uniform flow, sediment concentration at a given point of the water column is being balanced by gravity (right-hand side) trying to bring particles to the bed, and upward diffusion (left-hand side) which re-suspends the sediment[34]. Naturally, we cannot know $C(z)$ (i.e. solve the above differential equation) until we have an expression for D_{sz} (note that w_f is independent of z). And so we have another closure problem – remember Reynolds stresses? This should not be surprising; after all, the sediment diffusion coefficient does relate to turbulence: it is turbulence that suspends sediment (counteracting the influence of gravity). In fact, as we will see below, some similarities exist. There are several *models* for the diffusion coefficient D_{sz} (as with the eddie viscosity), none of which is perfect for all applications. For example, we may assume that

i) D_{sz} is some constant,

$$D_{sz} = D_0; \tag{15}$$

ii) D_{sz} varies linearly with the distance from the bed, z, as[35]

$$D_{sz} = \kappa u_* z, \tag{16}$$

where κ (≈ 0.4) is the von Kármán constant and u_* is the sear velocity; **or that**

[32]**IMPORTANT:** A couple of points. First, we are strictly talking here about the time-average concentration $\langle C \rangle$, but we avoid the angle brackets to keep the notation clearer. Second, also strictly speaking we are dealing here with **dispersion** rather than diffusion (in a molecular sense); in other words, the diffusion-like behaviour is due to the underlying flow patterns (mainly turbulence). However, because both diffusion and dispersion are modelled by the same type of equation, you will find in the literature that these terms are often used interchangeably. I keep to this convention in these notes.

[33]We will assume that $dC/dz \to 0$ when $C \to 0$, which gets rid of the integration constant.

[34]Show that D_{sz} must be non-negative (tip: think about the expected behaviour of the $C(z)$ profile).

[35]This is not completely arbitrary, it connects with the Prandtl eddie model for turbulence discussed previously in Open Channel Hydrodynamics.

iii) D_{sz} follows a parabolic profile throughout the water depth, h, according to

$$D_{sz} = \kappa u_* z \left(1 - \frac{z}{h}\right). \tag{17}$$

Use of the three models for D_{sz} (constant, linear and parabolic)[36] in (14), yields, respectively, the following expressions[37] for $C(z)$:

$$C(z) = C_0 \exp\left(-\frac{w_f}{D_0} z\right), \tag{18}$$

$$C(z) = C_0 \left(\frac{z}{z_0}\right)^{-\frac{w_f}{\kappa u_*}}, \tag{19}$$

and

$$C(z) = C_0 \left[\left(\frac{z}{h-z}\right)\left(\frac{h-z_0}{z_0}\right)\right]^{-\frac{w_f}{\kappa u_*}}, \tag{20}$$

where $C_0 = C(z = z_0)$ is some reference concentration at a reference height z_0 (typically, the near-bed concentration is used). The exponent $w_f/(\kappa u_*)$ is called the **Rouse number**, after the 20$^{\text{th}}$ century American fluid dynamicist Hunter Rouse, and is clearly of great importance in determining the concentration distribution. For instance, large values of the Rouse number imply a predominance of w_f over u_* (recall, u_* relates to turbulence), and thus weak or near-bed suspension will take place. The third expression above for $C(z)$ is called the **Rouse profile**.

The above expressions for the suspended concentration distribution, $C(z)$, are by no means unique. There exist many more, and analytical solutions for this problem continue to be published today. They are, however, typically employed in engineering practice. Fig. 7 illustrates a comparison between the profiles obtained by using linear and parabolic (Rouse profile) approximations for D_{sz}, for a Rouse number of 0.671 (high concentration, often known as 'wash load').

Finally, if one needs to know the total load of sediment being mobilised in suspension by the flow (per unit channel width), q_s, all that needs to be done is integrate:

$$q_s = \int_{z_0}^{h} C(z) u_s(z) dz. \tag{21}$$

If theoretical expressions for $C(z)$ and $u_s(z)$ are employed (for the latter, for instance, the law of the wall could be invoked, at least for the near-bed region and at low concentrations[38]), the above expression may have an analytical solution. Alternatively, there exist empirical formulae for the **suspended** or **total load** based on parameters more easily obtained in field or numerical simulations, such as the depth-average flow (streamwise) velocity.

[36] Sketch the three models, showing $D_{sz}(z)/(\kappa u_* h)$ in the $x-$axis and z/h in the $y-$axis.

[37] Obtain each of these expressions for $C(z)$.

[38] The low-concentration assumption is very important, for at high concentrations you can no longer assume that the sediment does not influence the local hydrodynamics (it is no longer a passive scalar), and very different theories apply (e.g. treating the hyper-concentrated water-sediment mixture as a non-Newtonian fluid).

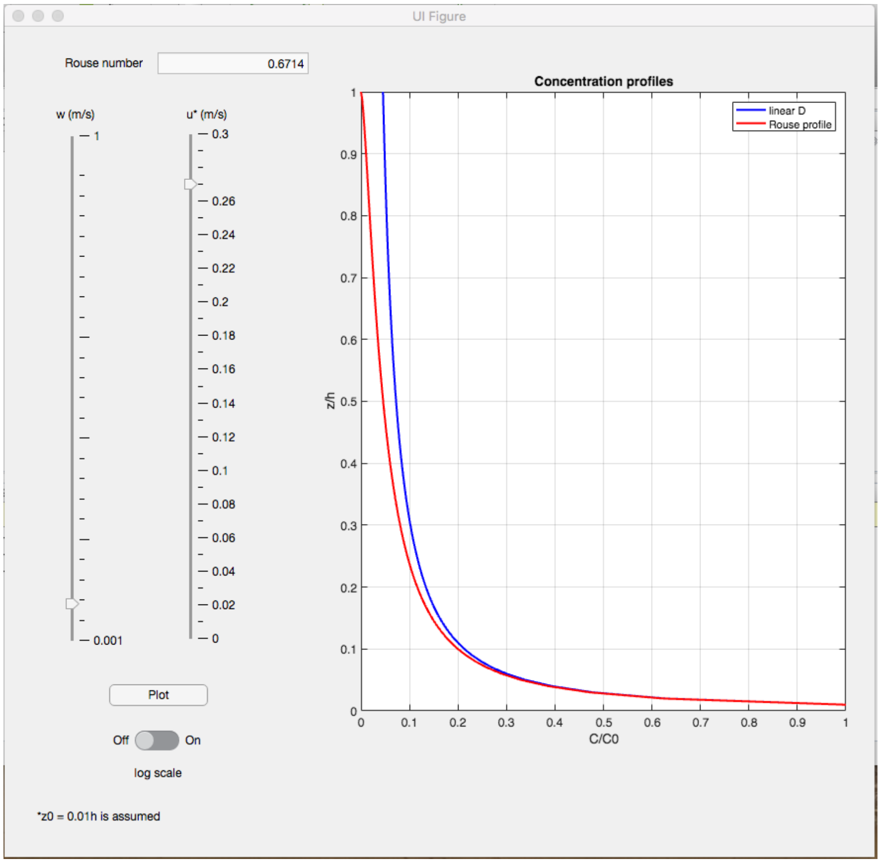

Figure 7: Comparison between concentration profiles predicted for linear (blue line) and parabolic (red line) approximations for D_{sz}, for a Rouse number of 0.671. Screenshot from Matlab app provided as additional material.

5 The bed evolution (Exner) equation

Up to this point, we have followed grains of sediment from the moment they start moving from their resting position on the bed, to the stage where they are transported by the flow, either as bedload or as suspended load. But we have ignored the fact that by taking sediment from the bed, the bed shape or **morphology** must change. The dynamics of a changing bed morphology are called **morphodynamics**. The importance of understanding the temporal evolution of the bed morphology will become clearer in the next section and coming parts of the module, but you may guess a few problems where this may be a crucial aspect: siltation of dams, dredging of rivers, scour around bridge piers, coastal erosion, etc. In any case, here we will focus on the general equation governing the bed change.

When the grains of sediment originally resting on the bed are picked up by the flow, they are transported by the latter until they eventually resettle onto the bed somewhere downstream. The important thing here is that the grains *go somewhere*, and so are obviously not lost. In other words, *bed sediment mass is conserved*. Therefore, to track the evolution of the bed level, it makes sense to apply conservation of sediment mass to a control volume located at the bed level. Focusing on the 1D case for simplicity, it is clear that sediment may enter the control volume either by bedload from the upstream end or be deposited from above; and sediment may leave the volume via bedload at the downstream end or by being eroded and suspended into the column of water[39]. Any deficit or surplus between the sediment entering and leaving the control volume should be reflected in the local change of bed level z_b according to:

$$\frac{\partial z_b}{\partial t} + \frac{1}{(1-\varepsilon_p)}\frac{\partial q_b}{\partial x} = e_s, \tag{22}$$

where ε_p is the bed porosity, typically taken as constant[40]; and e_s is the net flux of deposited sediment, i.e. the difference between the rate of sediment settling and the rate of sediment eroded and suspended into the column of fluid. The term $\partial q_b/\partial x$ represents the deficit or surplus that may exist between the bedload sediment leaving and entering our control volume. Morpological changes of engineering importance are usually dominated by bedload transport, such that the right-hand side of the above equation is often neglected[41]. Generalising to bedload in the two horizontal directions, we have

$$\frac{\partial z_b}{\partial t} = -\frac{1}{(1-\varepsilon_p)}\nabla\cdot\mathbf{q}_b, \tag{23}$$

where $\mathbf{q}_b = (q_{bx}, q_{by})$ is the bedload vector composed of bedload rates q_{bx} and q_{by} in the x and y directions, respectively. Thus, the above equation, also known as **Exner equation**, may also be written as

$$\frac{\partial z_b}{\partial t} = -\frac{1}{(1-\varepsilon_p)}\left(\frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y}\right). \tag{24}$$

Exner equation predicts the *local change in bed elevation* due to any *local gradient in the bedload transport*. And this is all we need to predict the overall evolution of any open channel bed subject to bedload, provided, of course, that we also know the local hydrodynamics (which dictate $\mathbf{q}_b$; see §7). The next section will show how this equation may be used to predict the evolution of bedforms.

6 Bedforms

If you have seen an exposed river bed, or a beach at low tide, you will have noticed that alluvial beds are typically not flat, but rather rich in certain *wavy features* (see fig. 8). These features are collectively known as **bedforms** (or due to their wavy/rhythmic patterns, also as **sandwaves**), and are ubiquitous to natural streams. Bedforms, their classification and mechanics, are not only fascinating from a scientific perspective, but are also very relevant in engineering practice. For instance, they effectively increase the flow resistance, they migrate and may interact with engineered structures (e.g. a bridge pier) and, for large forms, the local depth variations may affect navigation.

[39] Sketch this.

[40] Note that the term $(1-\varepsilon_p)$ may also be replaced by the bed sediment concentration or **packing fraction**, but the porosity is conventionally preferred (a porosity = 1 means a concentration = 0, and vice versa). Usually, $0.45 < \varepsilon_p < 0.75$, with $\varepsilon_p = 0.64$ being a good estimate for spherical grains.

[41] This also obeys historical reasons. Western science on open channel morphodynamics originally developed mainly in countries with rivers presenting relatively coarse sediment (Switzerland, UK, USA, Canada), which are dominated by bedload. But this is not true for other important rivers in the world, such as the suspension-dominated Yellow river in China, where many advances on the topic have also been made. Some Chinese researchers in the topic even go as far as to say that bedload is nothing but a limit case of suspended load. There is of course some arbitrariness to the definition of bedload and suspended load.

We will now look at some important aspects of bedforms and employ Exner equation to understand some of their mechanics.

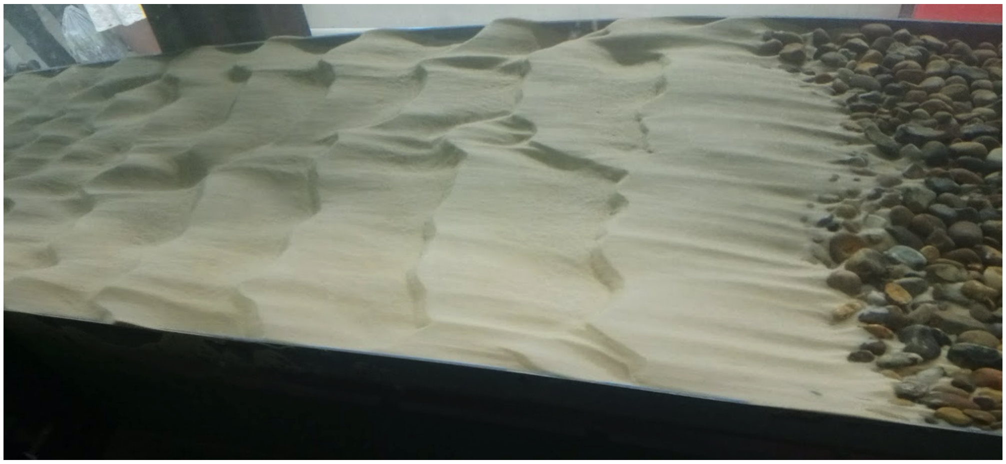

Figure 8: Bedforms generated during an experiment with sand bed at Boldrewood Campus, Univ. of Southampton. Flow was from right to left.

Sandwaves vary greatly in size, as shown in fig. 9. At the small-scale extreme of the spectrum, there are **ripples**, which typically have wavelengths and heights of centimetres and are associated with fine sediment ($D < 1$ mm). On the other side of the spectrum, **dunes** can have wavelengths of tens of metres and heights of metres. Interestingly, ripples may commonly be found superimposed in dunes. Various studies have related, via empirical expressions, the geometric characteristics of sandwaves (wavelength and height[42]) to relevant hydraulic parameters, such as depth and velocity. As usual, we will not focus here on said empirical expressions, but will rather highlight some conceptual aspects. For instance, it is worth remarking that the shape of bedforms induces flow separation in the downstream face (or lee side), where a large gradient in the bed slope is found, which results in a pressure drop at the lee side of the form (see fig. 9). In other words, (form) drag is being induced. This drag 'felt' by the sandwave is the same in magnitude that the flow experiences in the opposite direction. And thus, additional resistance (to that experienced by friction with an otherwise flat bed) is created. As you may expect, quantifying the enhanced resistance due to bedforms is not something that can be done with purely theoretical considerations, and so empiricism comes to the rescue once more. Many authors have proposed expressions to estimate, for instance, the enhanced roughness (or Manning's coefficient) due to the prescence of bedforms as a function of the latter's geometric characteristics. A bedform-induced shear stress τ'' may then be added to the 'normal' bed shear stress τ' that would be computed for a flat bed to yield an effective bed shear stress τ_b; i.e.

$$\tau_b = \tau' + \tau''. \tag{25}$$

The genesis of sandwaves also depends on the flow velocity, as one may expect. Ripples, for instance, are associated with flow velocities near the threshold of motion. Increase the flow velocity, and ripples give way to dunes (which sometimes have ripples superimposed). Increase the velocity further and the dunes will be washed away, leading to a flat bed. But for still higher flow velocities, another interesting type of bedform appears: **antidunes**. To see why antidunes are interesting, first we need to look at another aspect of sandwaves, namely, their migration.

[42] In fig. 8, identify the wavelength and height of the bedforms shown.

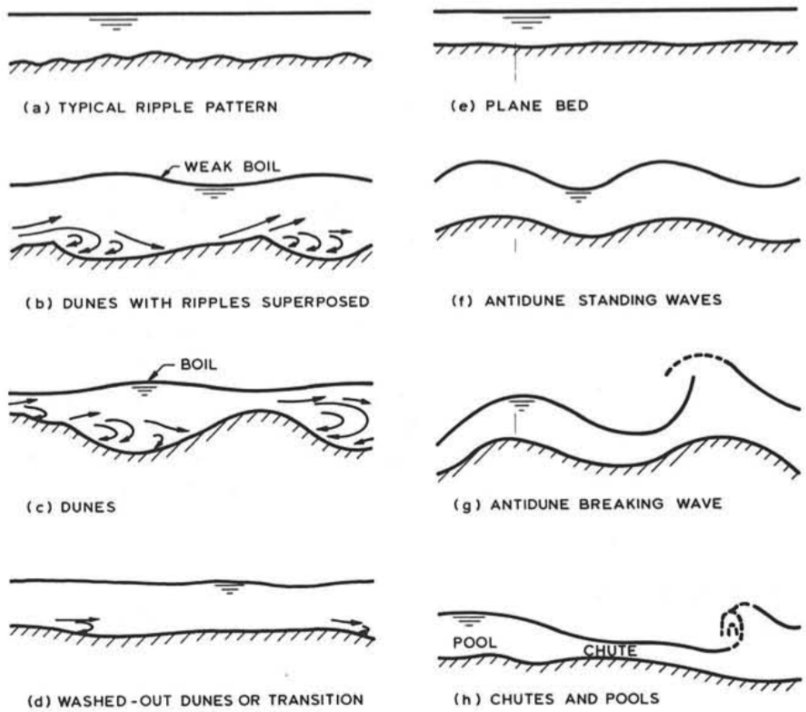

Figure 9: Different types of bedforms or sandwaves. [Taken from Jansen, P. P. *Principles of River Engineering: the Non-Tidal Alluvial River*. Delftse Uitgevers Maatschappij, 1994.]

For a given flow, the loose sediment in the bed gives rise to the forms discussed above, until a 'stable' shape is achieved (such as that shown in figs. 8 and 11). However, said beforms are not 'frozen' in space, but rather migrate. To understand how, it is convenient to consider the idealised 2D scenario[43] depicted in fig. 10, where an initial bedform is subject to a steady, subcritical current of fixed discharge and free surface level[44] (*not* water depth). It is clear that the flow (depth-averaged) velocity, U, increases towards the top of the form, and then decreases again in the lee side until attaining again its initial upstream value. Assume that U is large enough to mobilise sediment everywhere in the channel. Then, at an intuitive level, you can see that in parts of the bed where the fluid accelerates (the upstream or stoss side) net erosion will take place, whereas in those parts where the flow deaccelerates (the lee side), a tendency towards deposition will dominate. (We will get back to this problem later with a more quantitative, Exner-based, approach.) Since the flow upstream is steady, the result is a net streamwise migration, which is confirmed by field and laboratory observations of dunes (as shown in fig. 11). Just as with water waves, the velocity at which the sandwaves migrate is called their **celerity**, or **migration speed**. The celerity of real bedforms depends on several factors and typically requires use of empirical formulae and *in situ* measurements (or may be based purely on the latter) – a dune in an estuary, for instance, may migrate some 0.5 m in one day. However, for certain (very) ideal conditions[45], theoretical treatment

[43]Idealised in the sense that every real scenario will be 3D in nature, although several problems can be treated as 2D to highlight important aspects of the phenomenon.

[44]This is called the **rigid-lid assumption**, and it is an idealisation because, as you may remember from introductory hydraulics, for subcritical flow a depression in the surface level is expected to accompany a local increase in bed elevation (the step problem). However, the rigid-lid assumption is a good approximation for small values of the Froude number and small bedform height relative to the water depth (as you can see for yourself in fig. 10).

[45]Such as a *small* bedform in a wide rectangular channel, where bedload transport may be expressed as $q_b = aU^b$,

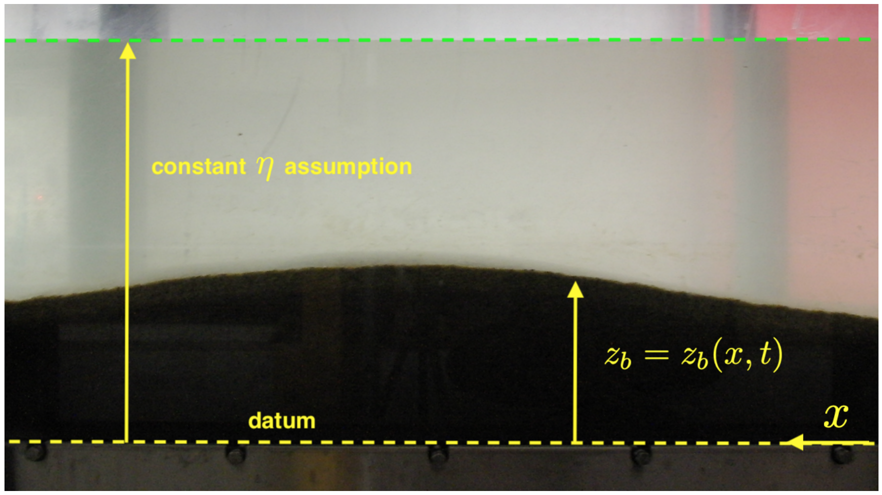

Figure 10: Initial state of an erodible hump subject to a subcritical, steady current (from right to left). Green dashed line illustrates the rigid-lid assumption. Experiment carried out at Boldrewood Campus, UoS.

permits derivation of the following estimate for the celerity of a bedform, c_b:

$$c_b = \frac{\alpha b a U^b}{h} \frac{1}{1 - F^2}, \tag{26}$$

where α, a and b are some empirical positive constants (see footnote 45), h is the water depth, and $F \equiv U/\sqrt{gh}$ is the Froude number. Of course, this equation is not defined for $F = 1$.

Now, let us analyse the above expression. As expected, the celerity or migration speed increases with the 'intensity' of the sediment transport rate (the term aU^b), and decreases with increasing water depth (because, for a given discharge per unit width, q, the velocity decreases with increasing depth: $U = q/h$). But the Froude number deserves especial attention. For $F < 1$, c_b increases with F and is always positive – this is in line with our intuitive findings above regarding the migration of a bedform in subcritical flow. But $F > 1$ (supercritical flow) predicts a negative value of c_b, and thus... upstream migration?! Counter-intuitive as it may sound, this *does occur* in nature, and is the reason why antidunes are so called (and why I told you they were interesting)[46]. In fact, there is a special case of beforms in between downstream migrating dunes and upstream migrating antidunes, namely, **standing waves**, which do not migrate at all.

Finally, let us return to Exner equation, which should reproduce the aspects of bedforms we have discussed so far. Consider again the problem depicted in fig. 10. Recall: the free surface is fixed at a constant value η (measured form a given datum), discharge per unit width is also constant and equal to q_0, the bed surface is given by $z_b(x,t)$, and for simplicity, bedload transport may be estimated as $q_b = aU^b$. Since the water depth is $h(x,t) = \eta - z_b(x,t)$, it follows that the depth-average velocity is given by $U(x,t) = q_0/(\eta - z_b(x,t))$. With this in mind, let us rewrite Exner

where a and b are some positive constants (usually, $b = 3$ is assumed).

[46] Explain with words, aided by sketches, how antidunes are formed from a qualitative viewpoint (tip: pay attention to the free surface profile for a supercritical flow).

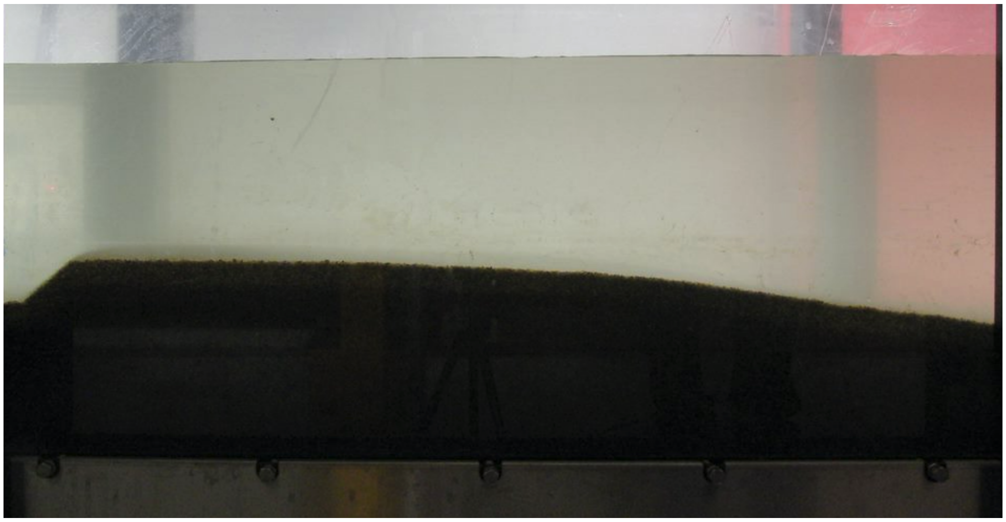

Figure 11: Evolution of the hump shown in fig. 10 after certain time. The form has evolved and migrated downstream. Flow is from right to left.

equation as[47]

$$\begin{aligned} & \frac{\partial z_b}{\partial t} + \frac{1}{(1-\varepsilon_p)}\frac{\partial q_b}{\partial x} \\ = \; & \frac{\partial z_b}{\partial t} + \frac{1}{(1-\varepsilon_p)}\frac{\partial q_b}{\partial z_b}\frac{\partial z_b}{\partial x} \\ = \; & \frac{\partial z_b}{\partial t} + \underbrace{\left[\frac{1}{(1-\varepsilon_p)} ab q_0^b \left(\eta - z_b\right)^{-(b+1)}\right]}_{\lambda(z_b)} \frac{\partial z_b}{\partial x} = 0. \end{aligned} \tag{27}$$

The equation above has the form of the advection equation (compare with eq. 11 with diffusion equal to zero), where the advection speed λ is the term in square brackets, which depends on several constants (a, b, q_0, ε_p and η) and z_b (this is therefore a nonlinear advection problem); i.e. $\lambda = \lambda(z_b)$. This means that points in different locations on the bed surface $z_b(x,t)$ will be *advected* at different speeds λ, depending on the height z_b from the datum, with points near the crest of the bedform being advected at faster rates than points near the base[48]. We can solve the above equation either analytically or numerically; in any case, the evolved bed will show the well-known shape of a dune (mild slope of upstream face and steep lee side), despite the fact that the original form was symmetrical (compare figs. 10 and 11). This is further verified in fig. 12, where the above equation has been solved analytically for the case of a perfectly symmetrical initial bedform.

Returning once more to Exner equation in the form of

$$\frac{\partial z_b}{\partial t} = -\frac{1}{(1-\varepsilon_p)}\frac{\partial q_b(U)}{\partial x}, \tag{28}$$

[47] I am taking some mathematical liberties here to make a point. What I mean by these ‘liberties’ is that, for example, the term $\partial q_b/\partial z_b$ has no direct physical meaning. But I need this to write the equation as an advection one, for reasons that are about to become clear.

[48] Think about this. It makes sense because for the assumptions adopted, a larger value of z_b means a smaller value of h (recall, $h = \eta - z_b$, and η is constant), and thus a larger $U(= q_0/h)$; and $q_b \propto U^b$.

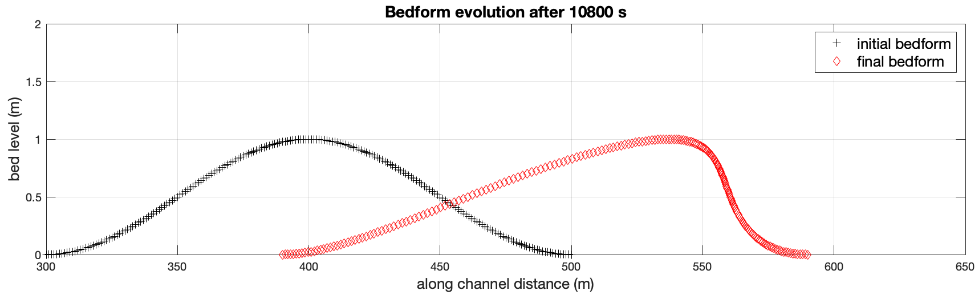

Figure 12: Evolution of an originally symmetrical bedform after 3 hrs of being subject to a steady current from left to right (analytical solution to eq. 27). The bedform changes shape while migrating downstream. The following parameters have been employed: $\eta = 10$ m; $q_0 = 10$ m^2/s; $q_b = 0.01U^3$ m^2/s; and $\varepsilon = 0.64$. Matlab code provided as additional material.

clearly shows that the bed will locally suffer erosion (accretion) if the bedload gradient is positive (negative). The bedload gradient is positive (negative) in the upstream (dowsntream) face of the bedform, where $\partial U/\partial x$ is positive (negative). This also shows quite neatly why bedform migration should be expected. The interesting behaviour of antidunes is also replicated by solving Exner equation, but since antidunes only occur in supercritical flows, the rigid-lid assumption referred to above (i.e. constant η) is no longer valid and Exner must be solved numerically alongside the flow hydrodynamics (e.g. by solving the Shallow Water Equations or the cross-section-mean equations), as discussed next.

7 Morphodynamic modelling

As we said at the beginning of these notes, the evolution of the bed morphology and the local hydrodynamics are intimately linked. The bed changes according to local sediment transport patterns, which in turn depend on the local flow velocity, which is in turn influenced by the bed (which is a boundary). Therefore, if we are interested in predicting (modelling) the morphodynamics of a given site (be it a river, an estuary or a coast), we must also model the hydrodynamics. For instance, if you are asked to predict the evolution of a navigation channel one year after it has been dredged, you should:

(i) obtain adequate field data (bathymetry and other boundary conditions, such as expected flow rates, tides, etc.); then

(ii) select a suitable hydrodynamic model for your problem (perhaps based on the Shallow Water Equations);

(iii) based on the simulated hydrodynamics, predict sediment transport patterns (after having selected –or developed/calibrated– a suitable sediment transport formula);

(iv) see how these sediment transport processes will affect the evolution of the bed (i.e. solve Exner equation); and, finally

(v) return to point (ii) (i.e. solve the hydrodynamics again) and carry on until you have reached the one-year prediction you were requested.

This process forms the basis of most existing morphodynamic models[49], and is illustrated in fig. 13.

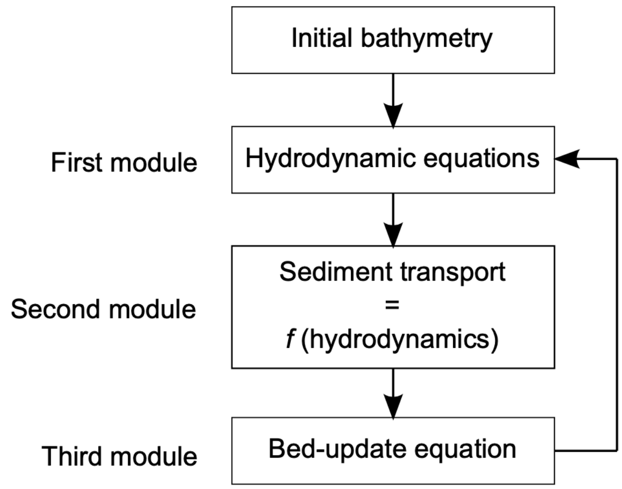

Figure 13: General structure of a typical morphodynamic model.

An illustration of the results from a morphodynamic model can be seen in fig. 14. Here, we want to know how the channel entrance to a harbour (initial bathymetry) will respond to a given combination of river discharge, tides and waves (hence, the hydrodynamic model will need to account for the influence of short-period water waves). The result of the hydrodynamic model is a time-varying depth-averaged 2D velocity vector field. This field is in turn employed to calculate a sediment transport field, whose gradient leads to instantaneous, local bed changes (Exner equation). The updated bed is then fed back into the hydrodynamic model, and the cycles repeats. Fig. 14C shows the erosion/deposition patterns (i.e. the difference between the final and the initial bathymetry) after a 50 days simulation.

We will conclude these notes with a couple of important remarks about morphodynamic models. First of all, it should be clear to you that the role played by the empirical expression you invoke to calculate sediment transport rates is crucial (this is *the* link between Exner equation and the hydrodynamic model). This is not to be understated. Research has shown that, just by changing the selection of empirical sediment transport formula (keeping everything else the same), the uncertainty in, say, the predicted formation time of certain bed patters can be in the order of tens of years! A real-world example of why this matters is provided by the recent, pioneering project of the Sand Engine in The Netherlands: a mega beach nourishment scheme designed to 'build with nature'[50]. Engineers and researchers investigating the potential useful life of this nourishment scheme (given of course by its morphodynamic evolution) disagree by tens of years, with significant implications for the associated costs: do we need to carry out another *huge* beach nourishment in 20 or 50 years? (I will let you speculate how such an uncertainty translates into costs in a 70 million Euro project) Another complicating factor of morphodynamic modelling is the large mismatch

[49] This is not the only possible way of predicting the morphodynamics of an open channel, but it is by far the most common. Other approaches include two-phase (water-sediment) and multi-layer (e.g. bed-bedload-suspension) models, which are however beyond the scope of this module.

[50] This is a prominent example of the so-called 'nature-based engineering' paradigm shift; learn more at https://dezandmotor.nl/en/.

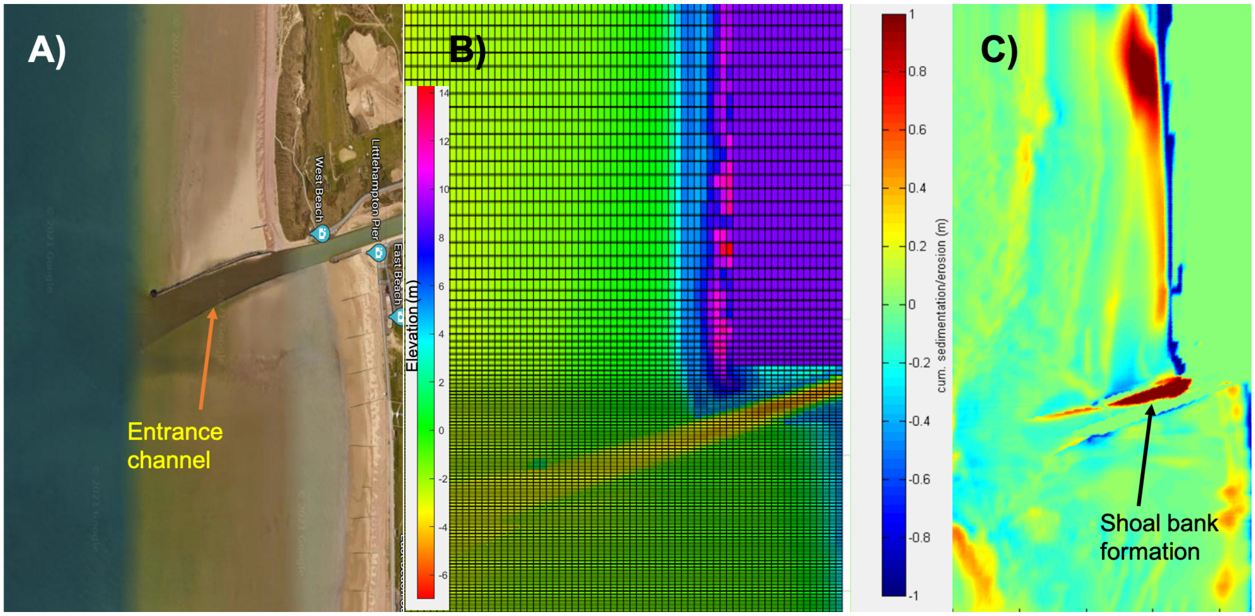

Figure 14: Morphodynamic modelling of the channel entrance to Littlehampton harbour, West Sussex. A) Satellite image of the site. B) Initial digital elevation map or bathymetry, which serves as initial condition for the model. C) Difference between final (after a 50-days simulation) and initial bathymetry, showing erosion/deposition patterns; pay attention to the shoal bank formation in the channel, which affects navigation into the harbour. (This is was part of a previous Group Design Project.)

between the time scales associated with the hydrodynamics and the morphodynamics, which are different by several orders of magnitude. In other words, the time-evolution of flow characteristics (depth, velocity) is typically measured in seconds, but morphological changes are usually only perceivable after hours or days. This mismatch continues to represent challenges to the engineering and scientific community, especially when interested in predicting the long-term evolution of the bed morphology.